

PURDUE UNIVERSITY MATH DEPARTMENT
PROBLEM OF THE WEEK
FALL 2011, PROBLEM 3

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Problem There are nine points in the interior of a cube of side 1. Show that at least two of the points are less than $\sqrt{3}/2$ apart.

Can $\sqrt{3}/2$ be replaced by a smaller number?

Solution Divide the cube into 8 congruent small cubes, each with side length $1/2$. By the pigeonhole principle, there exists a small cube containing at least two points. The maximum distance between two points in a small cube is $\sqrt{3}/2$, since this is the distance between opposite corners.

We cannot replace $\sqrt{3}/2$ with a smaller number because this upper bound is achieved with equality when eight points are placed on the corners of the original cube and a ninth point is placed in the center of the cube. □

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