

PURDUE UNIVERSITY MATH DEPARTMENT
PROBLEM OF THE WEEK
FALL 2011, PROBLEM 6

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Problem Given nine lattice points in space, show that there is an interior lattice point on at least one segment joining a pair of them.

Solution Each coordinate of a lattice point in space can be either even or odd, and thus there are $2^3 = 8$ possible assignments of parity for a lattice point. Given nine lattice points, by the pigeonhole principle there exists at least two lattice points (x_i, y_i) and (x_j, y_j) with the same parity. Their midpoint is then a lattice point, because $\frac{x_i+x_j}{2}$ and $\frac{y_i+y_j}{2}$ are both integers. \square

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