

PURDUE UNIVERSITY MATH DEPARTMENT
PROBLEM OF THE WEEK
SPRING 2011, PROBLEM 11

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Problem Prove or disprove that a rook can move from one corner to the diagonally opposite corner of a chessboard and cover every square exactly once.

Solution A chessboard has 64 squares which can be colored black and white in a checkerboard fashion. We may think of the rook as moving in one-step horizontal and vertical increments across this board. Because every square must be visited exactly once, the rook must take exactly 63 steps to get from one corner to the diagonally opposite corner. Since this is an odd number of steps, the rook's final square and initial square must be of different colors. However, this contradicts the fact that diagonally opposite squares on a chessboard have the same color. Therefore a Hamiltonian path from one corner to the other does not exist. \square

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