PURDUE UNIVERSITY MATH DEPARTMENT
PROBLEM OF THE WEEK
SPRING 2012, PROBLEM 1

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Problem Three hundred men sit around a circular table. The men are numbered 1 through 300 and each man has two neighbors. (The neighbors of 1 are 2 and 300, and the neighbors of 300 are 1 and 299.)

There are three hundred waiters, also numbered from 1 to 300. Each waiter has an urn containing three balls, one lettered L, and C and one R. Each waiter y draws a ball at random from his urn and if the ball is lettered L, delivers a dessert to the man to the left of man y. If the letter is C man y gets the dessert, and if the letter is R the man to the right of man y gets the dessert. Call a man lucky if he gets three desserts. Find the greatest possible number of lucky men, and the probability that this many men are lucky.

Solution Since there are 300 desserts in total, and a man is lucky if and only if he receives 3 desserts, there can be no more than 100 lucky men. We now show that this is achievable. For convenience of notation, let us renumber the waiters from 0 to 299. Now observe that man i is lucky if and only if

(1) waiter $i-1 \bmod 300$ draws an R,
(2) waiter $i \bmod 300$ draws an C, and
(3) waiter $i+1 \bmod 300$ draws an L,

since there is no other way for man i to receive 3 desserts. Therefore, any of the following three waiter configurations

waiter number	0	1	2	3	4	5	...	297	298	299
drawn ball	R	C	L	R	C	L	...	R	C	L

waiter number	0	1	2	3	4	5	...	297	298	299
drawn ball	L	R	C	L	R	C	...	L	R	C

waiter number	0	1	2	3	4	5	...	297	298	299
drawn ball	C	L	R	C	L	R	...	C	L	R

will result in the maximal number of lucky men (100), and since 300 is a multiple of 3, no other configurations can achieve this. The probability of having 100 lucky men is thus

$$\frac{3}{3^{300}} = \frac{1}{3^{299}}.$$

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