

PURDUE UNIVERSITY MATH DEPARTMENT
PROBLEM OF THE WEEK
SPRING 2013, PROBLEM 1

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Problem Show that every set of $n + 1$ positive integers, chosen from a set of $2n$ consecutive integers, contains at least one pair of relatively prime numbers.

Solution Let T denote the set of $2n$ consecutive integers, and let S denote an arbitrary set of $n + 1$ positive integers drawn from it. Recall that two consecutive integers are always coprime, since if p divides both k and $k + 1$, then p must also divide $(k + 1) - k = 1$, so p can only be 1. Consequently, it suffices to argue that S contains at least one pair of consecutive integers. This follows from the pigeonhole principle by setting the $n + 1$ numbers in S as the pigeons, and the n nonoverlapping pairs of consecutive integers in T as the holes. \square

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